A Tool for the Coverability Problems in Petri Nets

MSR 2019
Alain Finkel, Serge Haddad, Igor Khmelnitsky
Coverability Problems

Given a Petri net $(N, m/\text{zero.pnum})$, with $m/\text{zero.pnum} \in N^P$.

Coverability: Given a target marking $m_t \in N^P$, $\exists \sigma m/\text{zero.pnum} \sigma \rightarrow m' \geq m_t$.

Coverability set: Build $\text{Cover}(N, m/\text{zero.pnum}) \subseteq N^P_\omega$ such that:

$m \in N^P$ is coverable $\iff m_\omega \in \text{Cover}(N, m/\text{zero.pnum}), m \leq m_\omega$.
Coverability Problems

Given an Petri net \((\mathcal{N}, m_0)\), with \(m_0 \in \mathbb{N}^P\).
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Given an Petri net \((\mathcal{N}, m_0)\), with \(m_0 \in \mathbb{N}^P\).

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\[
m \in \mathbb{N}^P \text{ is coverable } \iff m_\omega \in Clover(\mathcal{N}, m_0), \ m \leq m_\omega
\]
Coverability set
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Early works

• Karp-Miller (one.pnum/nine.pnum/six.pnum/nine.pnum)
• Finkel (AF) (one.pnum/nine.pnum/nine.pnum/three.pnum)

Looking for minimal space (two.pnum/zero.pnum/zero.pnum/five.pnum)

Recent works

• Geeraets et al. (GR) (two.pnum/zero.pnum/one.pnum/zero.pnum)
• Reynier et al. (MP) (two.pnum/zero.pnum/one.pnum/three.pnum)

Fixing Finkel

• Valmari et al. (VH) (two.pnum/zero.pnum/one.pnum/four.pnum/one.pnum/six.pnum)

Looking for minimal time (one.pnum). The Petri net mesh/two.pnumx/two.pnum from [two.pnum/six.pnum].
Coverability set

Early works

- Karp-Miller (1969)
Coverability set

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  Intractable
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  Looking for minimal time
MinCov for Coverability Set
MinCov for Coverability Set

Algorithmic features

- A fix for Finkel’s algorithm based on accelerations
MinCov for Coverability Set

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- A fix for Finkel’s algorithm based on accelerations
- A theoretical bound on the number of accelerations
MinCov for Coverability Set

Algorithmic features

- A fix for Finkel’s algorithm based on accelerations
- A theoretical bound on the number of accelerations

Implementation features

- Written in Python, ≈ 2000 lines.
- Can be found in https://github.com/IgorKhm/MinCov
## Benchmarks

### 123 benchmarks (literature)

<table>
<thead>
<tr>
<th></th>
<th>T/O&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Time</th>
<th>#Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinCov</td>
<td>16</td>
<td>18127</td>
<td>48218</td>
</tr>
<tr>
<td>VH</td>
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<td>75225</td>
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<td>MP</td>
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<td>23904</td>
<td>478681</td>
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<tr>
<td>GR</td>
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<td>47089</td>
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<tr>
<td>AF</td>
<td>19</td>
<td>19223</td>
<td>45660</td>
</tr>
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</table>

1. Timeout after 900 seconds.

### 100 benchmarks (random)

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<tbody>
<tr>
<td>MinCov</td>
<td>14</td>
<td>13989</td>
<td>61164</td>
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<td>VH</td>
<td>15</td>
<td>13692</td>
<td>208134</td>
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<td>MP</td>
<td>21</td>
<td>21726</td>
<td>755129</td>
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<tr>
<td>GR</td>
<td>80</td>
<td>74767</td>
<td>N/A</td>
</tr>
<tr>
<td>AF</td>
<td>16</td>
<td>15888</td>
<td>63275</td>
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Coverability problem
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Blondin et al. (qCover) (2016)

Combining backward exploration with forward over-approximation
Coverability problem

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<td></td>
<td>Time</td>
<td>T/O</td>
<td>Time</td>
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<tr>
<td>MinCov</td>
<td>1754 1</td>
<td>51323 53</td>
<td>54</td>
</tr>
<tr>
<td>qCover</td>
<td>26467 26</td>
<td>11865 11</td>
<td>37</td>
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Complementary tools!
Coverability problem

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<td>11865</td>
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<tr>
<td>MinCov</td>
<td></td>
<td>qCover(^1)</td>
<td>1841</td>
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\(^1\) Time(MinCov || qCover) = 2 \min (\text{Time(MinCov)}, \text{Time(qCover)}).
Thank you!

See you at the tool demonstration!