Algorithm for Controlling the Transient Behavior of Controlled Generalized Batches Petri Nets

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Objective

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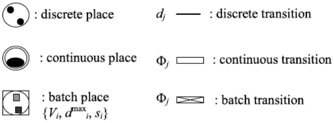
Compute a control trajectory to reach a target steady state from a given initial state in hybrid /discrete event systems.

Generalized batches Petri nets

A generalized batches Petri net (GBPN) is a 6-tuple $N = (P, T, Pre, Post, \gamma, Time)$ where:

- $P = P^D \cup P^C \cup P^B$
- $T = T^D \cup T^C \cup T^B$
- Pre and $Post$: $(P^D \times T \to \mathbb{N}) \cup ((P^C \cup P^B) \times T \to \mathbb{R}_{\geq 0})$
- $\gamma : P^B \to \mathbb{R}^3_{>0}$, $\gamma(p_i) = (V_i, d_i^{\max}, s_i)$
- $Time : T \to \mathbb{R}_{\geq 0}$, $Time(t_j) = d_j$ if $t_j \in T^D$; $Time(t_j) = \Phi_j$ if $t_j \in T^C \cup T^B$



Assumption 1: No discrete nodes ($P^D = T^D = \emptyset$) and no continuous transitions ($T^C = \emptyset$).

I. Demongodin. Generalised batches Petri net: hybrid model for high speed systems with variable delays. DEDS. 2001

Batch, marking and marking quantity

A batch β_k at time τ is defined by a triple $\beta_k(\tau) = (l_k(\tau), d_k(\tau), x_k(\tau))$, where $l_k(\tau) \in \mathbb{R}_{\geq 0}$ is the length, $d_k(\tau) \in \mathbb{R}_{\geq 0}$ is the density and $x_k(\tau) \in \mathbb{R}_{\geq 0}$ is the head position.

The marking of a GBPN at time τ, $\boldsymbol{m}(\tau) = [m_1(\tau)m_2(\tau)\dots m_m(\tau)]^T$, is a function that assigns to each continuous place a nonnegative real number and assigns to each batch place a series of batches $m_i(\tau) = \{\beta_1(\tau), \dots, \beta_r(\tau)\}$.

The marking quantity vector $\boldsymbol{q} = \mu(\boldsymbol{m}) \in \mathbb{R}^m$ associated with a marking $\boldsymbol{m}$ is defined as: $q_i = \begin{cases} m_i & \text{if } p_i \in P^C \\ \sum_{k=1}^{r} l_k \cdot d_k & \text{if } p_i \in P^B. \end{cases}$

Instantaneous firing flow(IFF)

The instantaneous firing flow (IFF) vector at time τ is denoted as $\boldsymbol{\varphi}(\tau) \in \mathbb{R}^{|T|}$, where $\varphi_j(\tau) \leq \Phi_j$ represents the firing quantity of transition t_j by time unit.
The input flow (resp., output flow) of a batch or continuous place p_i at time τ is the sum of all flows entering (resp., leaving) the place and can be written, respectively, as:

- $\phi_i^{\text{in}}(\tau) = Post(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$,
- $\phi_i^{\text{out}}(\tau) = Pre(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$.

Remark:Between timed events, $\boldsymbol{\varphi}$, $\boldsymbol{\phi}^{\text{in}}$ and $\boldsymbol{\phi}^{\text{out}}$ are constants.

The fundamental equation is

$$\boldsymbol{q}(\tau) = \boldsymbol{q}(\tau_0) + \boldsymbol{C} \cdot \int_{\tau_0}^{\tau} \boldsymbol{\varphi}(\rho) d\rho.$$

Steady state

Steady state with assigned transfer speed

Let $\langle N, \boldsymbol{m}_0 \rangle$ be a GBPN system with $P^D = T^D = \emptyset$. The net is in a steady state (SS) at time τ_s if for $\tau \geq \tau_s$ the marking $\boldsymbol{m}^s$ and the instantaneous firing flow vector φ^s remain constant. Thus a steady state is defined by a pair $(\boldsymbol{m}^s, \varphi^s)$.

$(\boldsymbol{q}^s, \varphi^s)$ can be obtained by considering the following constraint set :

$$
\begin{cases}
\text{(a)} & \mathbf{0} \leq \boldsymbol{y} \leq \boldsymbol{\Phi} \\
\text{(b)} & Q_i \geq q_i \geq \mathrm{Pre}\,(p_i, \cdot) \cdot \boldsymbol{y} \cdot s_i / V_i \quad (\forall p_i \in P^B) \\
\text{(c)} & Post\,(p_i, \cdot) \cdot \boldsymbol{y} \leq V_i \cdot d_i^{\max} \quad (\forall p_i \in P^B) \\
\text{(d)} & \mathbf{C} \cdot \boldsymbol{y} = \mathbf{0} \\
\text{(e)} & \boldsymbol{q} \in RQ\,(\mathrm{N}, \boldsymbol{m}_0)
\end{cases}
$$

where $\boldsymbol{q} \in \mathbb{R}^m$ and $\boldsymbol{y} \in \mathbb{R}^n$ are variables of the constraint sets, $RQ\,(\mathrm{N}, \boldsymbol{m}_0)$ denotes the reachable marking quantity set.

Proposition

From q^s and φ^s, the regular marking m^s can be uniquely reconstructed, denoted $m^s = \nu(q^s, \varphi^s)$.

I. Demongodin and A. Giua. Dynamics and steady state analysis of controlled generalized batches Petri nets. NASH, 2014.

Controlled GBPNs and control trajectory

cGBPNs

The controlled GBPNs have the same syntax of GBPNs. The instantaneous firing flow of continuous and batch transitions and the transfer speed of batch places are control inputs that can be used to drive the evolution of the net.

Remark 1: The controlled firing flow vector of cGBPNs is denoted as $\boldsymbol{u}(\tau)$, with $0 \leq u_j(\tau) \leq \Phi_j$.

Remark 2: The transfer speed is assumed to be constant in this work.

Control trajectory

Given a cGBPN system $\langle N, \boldsymbol{m}_0 \rangle$, a control trajectory is given as $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots,$ $(\boldsymbol{u}^i, \tau_i), \cdots, (\boldsymbol{u}^n, \tau_n)$ such that the controlled firing flow vector $\boldsymbol{u}^i$ is applied at date τ_i until τ_{i+1}.

Control strategy

Given a cGBPN system $\langle N, \boldsymbol{m}_0 \rangle$ and a reachable steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$.

Assumption 2: Steady firing flow vector is positive ($\boldsymbol{\varphi}^s > \mathbf{0}$).

Control strategy

1. OFF: $u_j(\tau) = 0$ if t_j is not enabled or the marking quantity of one of its input places p_i is lower than its steady marking quantity $q_i(\tau) < q_i^s$.
2. ON: maximize $\boldsymbol{u}(\tau)$ for approaching the steady marking quantity vector $\boldsymbol{q}^s$.

$$\boldsymbol{q}^s(\tau) = \boldsymbol{q}^0(\tau_0) + \boldsymbol{C} \cdot \left(\int_{\tau_0}^{\tau_1} \boldsymbol{u}^0(\rho) d\rho + \cdots + \int_{\tau_{n-1}}^{\tau_n} \boldsymbol{u}^{n-1}(\rho) d\rho + \int_{\tau_n}^{\infty} \boldsymbol{u}^n(\rho) d\rho \right),$$

with $C \cdot \int_{\tau_n}^{\infty} \boldsymbol{u}^n(\rho) d\rho = 0$.

L. Wang, C. Mahulea, J. julvez, and Manuel Silva. On/off strategy based minimum-time control of continuous Petri nets. NASH, 2014.

Thanks for your attention!