Algorithm for Controlling the Transient Behavior of Controlled Generalized Batches Petri Nets

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Objective

Compute a control trajectory to reach a target steady state from a given initial state in hybrid / discrete event systems.
A generalized batches Petri net (GBPN) is a 6-tuple $N = (P, T, Pre, Post, \gamma, Time)$ where:

- $P = P^D \cup P^C \cup P^B$
- $T = T^D \cup T^C \cup T^B$
- $Pre$ and $Post : (P^D \times T \rightarrow \mathbb{N}) \cup ((P^C \cup P^B) \times T \rightarrow \mathbb{R}_{\geq 0})$
- $\gamma : P^B \rightarrow \mathbb{R}^3_{>0}, \gamma(p_i) = (V_i, d_{i}^{\text{max}}, s_i)$
- $Time : T \rightarrow \mathbb{R}_{\geq 0}, Time(t_j) = d_j$ if $t_j \in T^D$; $Time(t_j) = \Phi_j$ if $t_j \in T^C \cup T^B$

**Assumption 1:** No discrete nodes ($P^D = T^D = \emptyset$) and no continuous transitions ($T^C = \emptyset$).

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A batch $\beta_k$ at time $\tau$ is defined by a triple $\beta_k(\tau) = (l_k(\tau), d_k(\tau), x_k(\tau))$, where $l_k(\tau) \in \mathbb{R}_{\geq 0}$ is the length, $d_k(\tau) \in \mathbb{R}_{\geq 0}$ is the density and $x_k(\tau) \in \mathbb{R}_{\geq 0}$ is the head position.

The marking of a GBPN at time $\tau$, $m(\tau) = [m_1(\tau)m_2(\tau) \ldots m_m(\tau)]^T$, is a function that assigns to each continuous place a nonnegative real number and assigns to each batch place a series of batches $m_i(\tau) = \{\beta_1(\tau), \ldots, \beta_r(\tau)\}$.

The marking quantity vector $q = \mu(m) \in \mathbb{R}^m$ associated with a marking $m$ is defined as: $q_i = \begin{cases} m_i & \text{if } p_i \in P^C \\ \sum_{k=1}^{r} l_k \cdot d_k & \text{if } p_i \in P^B. \end{cases}$
The **instantaneous firing flow** (IFF) vector at time $\tau$ is denoted as $\varphi(\tau) \in \mathbb{R}^{|T|}$, where $\varphi_j(\tau) \leq \Phi_j$ represents the firing quantity of transition $t_j$ by time unit. The **input flow** (resp., **output flow**) of a batch or continuous place $p_i$ at time $\tau$ is the sum of all flows entering (resp., leaving) the place and can be written, respectively, as:

- $\phi_i^{\text{in}}(\tau) = \text{Post}(p_i, \cdot) \cdot \varphi(\tau)$,
- $\phi_i^{\text{out}}(\tau) = \text{Pre}(p_i, \cdot) \cdot \varphi(\tau)$.

**Remark:** Between timed events, $\varphi$, $\phi^{\text{in}}$ and $\phi^{\text{out}}$ are constants.

The **fundamental equation** is

$$q(\tau) = q(\tau_0) + C \cdot \int_{\tau_0}^{\tau} \varphi(\rho) d\rho.$$
Steady state with assigned transfer speed

Let $\langle N, m_0 \rangle$ be a GBPN system with $P^D = T^D = \emptyset$. The net is in a steady state (SS) at time $\tau_s$ if for $\tau \geq \tau_s$ the marking $m^s$ and the instantaneous firing flow vector $\varphi^s$ remain constant. Thus a steady state is defined by a pair $(m^s, \varphi^s)$.

$(q^s, \varphi^s)$ can be obtained by considering the following constraint set:

$$
\begin{align*}
(a) & \quad 0 \leq y \leq \Phi \\
(b) & \quad Q_i \geq q_i \geq \text{Pre} (p_i, \cdot) \cdot y \cdot s_i / V_i \quad (\forall p_i \in P^B) \\
(c) & \quad \text{Post} (p_i, \cdot) \cdot y \leq V_i \cdot d_i^{\text{max}} \quad (\forall p_i \in P^B) \\
(d) & \quad C \cdot y = 0 \\
(e) & \quad q \in RQ (N, m_0)
\end{align*}
$$

where $q \in \mathbb{R}^m$ and $y \in \mathbb{R}^n$ are variables of the constraint sets, $RQ (N, m_0)$ denotes the reachable marking quantity set.

**Proposition**

From $q^s$ and $\varphi^s$, the regular marking $m^s$ can be uniquely reconstructed, denoted $m^s = \nu(q^s, \varphi^s)$.
cGBPNs

The controlled GBPNs have the same syntax of GBPNs. The instantaneous firing flow of continuous and batch transitions and the transfer speed of batch places are control inputs that can be used to drive the evolution of the net.

Remark 1: The controlled firing flow vector of cGBPNs is denoted as $u(\tau)$, with $0 \leq u_j(\tau) \leq \Phi_j$.

Remark 2: The transfer speed is assumed to be constant in this work.

Control trajectory

Given a cGBPN system $\langle N, m_0 \rangle$, a control trajectory is given as $(u^0, \tau_0), (u^1, \tau_1), \cdots, (u^i, \tau_i), \cdots, (u^n, \tau_n)$ such that the controlled firing flow vector $u^i$ is applied at date $\tau_i$ until $\tau_{i+1}$. 
Given a cGBPN system \( \langle N, m_0 \rangle \) and a reachable steady state \((m^s, \varphi^s)\).

**Assumption 2:** Steady firing flow vector is positive \((\varphi^s > 0)\).

**Control strategy**

1. OFF: \( u_j(\tau) = 0 \) if \( t_j \) is not enabled or the marking quantity of one of its input places \( p_i \) is lower than its steady marking quantity \( q_i(\tau) < q_i^s \).

2. ON: maximize \( u(\tau) \) for approaching the steady marking quantity vector \( q^s \).

\[
q^s(\tau) = q^0(\tau_0) + C \cdot \left( \int_{\tau_0}^{\tau_1} u^0(\rho)d\rho + \cdots + \int_{\tau_{n-1}}^{\tau_n} u^{n-1}(\rho)d\rho + \int_{\tau_n}^{\infty} u^n(\rho)d\rho \right),
\]

with \( C \cdot \int_{\tau_n}^{\infty} u^n(\rho)d\rho = 0. \)

Thanks for your attention!